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# Meson spectrum in Regge phenomenology

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**Abstract.** Under the assumption that both light and heavy quarkonia populate approximately linear Regge trajectories with the requirements of additivity of intercepts and inverse slopes, the masses of different meson multiplets are estimated. The predictions derived from the quasi-linear Regge trajectories are in reasonable agreement with those given by many other references.

## 1 Introduction

The investigation of the meson spectrum is of great importance for better understanding the dynamics of the strong interactions, since the mesons are the ideal laboratory for the study of strong interactions in the strongly coupled non-perturbative regime [1]. According to the recent issue of the Review of Particle Physics [2], there are many mesons so far absent from the Meson Summary Table; therefore, for the sake of the completeness of meson spectrum, especially for the heavy meson spectrum, there is still a lot of work to be done both theoretically and experimentally.

Regge theory is concerned with the particle spectrum, the forces between the particles and the high energy behavior of the scattering amplitudes [3]. One of the most distinctive features of Regge theory is the Regge trajectory by which the mass and the spin of a hadron are related. Knowledge of the Regge trajectories is useful not only for spectral purposes, but also for many non-spectral purposes. The intercepts and slopes of the Regge trajectories are of fundamental importance in hadron physics [4].

To a large extent, our knowledge of meson spectrum is based on some phenomenological QCD motivated models. A series of recent papers by Anisovich et al. [5] show that meson states fit to the quasi-linear Regge trajectories with sufficiently good accuracy, although some suggestions that the realistic Regge trajectories could be non-linear exist [6,7].

In the analysis of [5], the Regge trajectories for heavy mesons were not involved. In the analysis of [8,9], the mass relation for heavy mesons was investigated in the quasi-linear Regge trajectory ansatz with the simplification that the Regge slopes in the light quark sector are the same for all the meson multiplets. With the help of the rich available experimental data, in the present work we shall extract the parameters of the quasi-linear Regge trajectories for both light and heavy meson states and estimate the masses of the states lying on these Regge trajectories. In our consideration, we do not constrain the Regge slope to be flavor-independent, and we shall adopt the argument adopted by [5] that the state with spin $J$ and its partners with the same quantum numbers but spin $J+2$, $J+4$, $\cdots$, rather than both the $J^{PC}$ and the $(J+1)^{-P,-C}$ states, populate the common quasi-linear Regge trajectory. The suggestion [9] that the Regge trajectories are not linear but rather have curvatures in the region of lower spin may be relevant to the usual assignment that both $J^{PC}$ and $(J+1)^{-P,-C}$ states belong to a common linear Regge trajectory. In fact, based on this assignment, if one tries to fit the $\pi$ $(0^{-+})$ to the linear trajectory on which the $b_1(1235)$ $(1^{+-})$ and $\pi_2(1670)$ $(2^{-+})$ lie, one can obtain that the mass of $\pi$ is about $0.696\,\mathrm{GeV}$, which is much higher than the experimental value of $0.135\,\mathrm{GeV}$ [2]. The argument that both $J^{PC}$ and $(J+1)^{-P,-C}$ states populate a common linear Regge trajectory is in fact based on the hypothesis of exchange-degeneracy of the Regge trajectories [3]. However, it has already been pointed out by Desgrolard et al. [10] that the hypothesis of exact exchange-degeneracy, even in its weak formulation, is not supported by the present data.

This paper is organized as follows. The parameters of the different Regge trajectories and the masses of the meson states lying on the different Regge trajectories are given in Sect. 2. The discussions of our results appear in Sect. 3. The summary and conclusion are presented in Sect. 4.

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## 2 The parameters of Regge trajectories and spectroscopy

By assuming the existence of the quasi-linear Regge trajectories for a meson multiplet, one can have

$$J = \alpha_{i\bar{i}'}(0) + \alpha'_{i\bar{i}'} M^2_{i\bar{i}'}, \tag{1}$$

where $i$ ($\bar{i}'$) refers to the quark (antiquark) flavor, $J$ and $M_{i\bar{i}'}$ are respectively the spin and mass of the $i\bar{i}'$ meson, $\alpha_{i\bar{i}'}(0)$ and $\alpha'_{i\bar{i}'}$ are respectively the intercept and slope of the trajectory on which the $i\bar{i}'$ meson lies. For a meson multiplet, the parameters for different flavors can be related by the following relations proposed in the literature:

(i) additivity of intercepts,

$$\alpha_{i\bar{i}}(0) + \alpha_{j\bar{j}}(0) = 2\alpha_{j\bar{i}}(0), \tag{2}$$

(ii) additivity of inverse slopes,

$$\frac{1}{\alpha'_{i\bar{i}}} + \frac{1}{\alpha'_{j\bar{j}}} = \frac{2}{\alpha'_{j\bar{i}}}, \tag{3}$$

(iii) factorization of slopes,

$$\alpha'_{i\bar{i}}\alpha'_{j\bar{j}} = (\alpha'_{j\bar{i}})^2. \tag{4}$$

The relation (2) has for a first time been derived for $u(d)$- and $s$-quarks in the dual-resonance model [11], and it is satisfied in two-dimensional QCD [12], the dual-analytic model [13], and the quark bremsstrahlung model [14]. Also, it saturates the inequalities for Regge trajectories [15] which follow from the $s$-channel unitarity condition. The relation (3) is derived based on a topological expansion and the $q\bar{q}$-string picture of hadrons [17], and the relation (4) follows from the factorization of residues of the $t$-channel poles [18]. The paper by Burakovsky et al. [16] shows that only additivity of inverse Regge slopes is consistent with the formal chiral and heavy quark limits for both mesons and baryons, and factorization of the Regge slopes, although consistent in the formal chiral limit, fails in the heavy quark limit. In our present work, we shall assume that the relations (2) and (3) are valid for the quasi-linear Regge trajectory.

Based on the quasi-linear Regge trajectory (1), together with the relations (2) and (3), we can construct the Regge trajectories. The starting point for constructing a meson Regge trajectory is the meson assignment. According to the argument that the state with spin $J$ and its partners with the same quantum numbers but spin $J+2$, $J+4$, $\cdots$ populate a common linear Regge trajectory [10], the meson assignment is shown in Table 1. In the following, $n$ denotes a $u$- or $d$-quark.

### 2.1 The $1\,^1S_0$ trajectories

For the $1\,^1S_0$ trajectories, inserting the masses of $\pi$, $\pi_2(1670)$, $K$, $K_2(1770)$, $\eta_c(1S)$, $D$, $\eta_b(1S)$ and $B$[1] into the following equations:

$$0 = \alpha_{n\bar{n}}(0) + \alpha'_{n\bar{n}} M^2_{\pi}, \tag{5}$$
$$2 = \alpha_{n\bar{n}}(0) + \alpha'_{n\bar{n}} M^2_{\pi_2(1670)}, \tag{6}$$
$$0 = \alpha_{n\bar{s}}(0) + \alpha'_{n\bar{s}} M^2_{K}, \tag{7}$$
$$2 = \alpha_{n\bar{s}}(0) + \alpha'_{n\bar{s}} M^2_{K_2(1770)}, \tag{8}$$
$$0 = \alpha_{c\bar{c}}(0) + \alpha'_{c\bar{c}} M^2_{\eta_c(1S)}, \tag{9}$$
$$0 = \alpha_{c\bar{n}}(0) + \alpha'_{c\bar{n}} M^2_{D}, \tag{10}$$
$$0 = \alpha_{b\bar{b}}(0) + \alpha'_{b\bar{b}} M^2_{\eta_b(1S)}, \tag{11}$$
$$0 = \alpha_{n\bar{b}}(0) + \alpha'_{n\bar{b}} M^2_{B}, \tag{12}$$

and with the help of the relations (2) and (3), one can extract the parameters of the $1\,^1S_0$ trajectories shown in Table 2.

Based on these parameters, the masses of the $J=0$, $J=2$ and $J=4$ states lying on the $1\,^1S_0$ trajectories can be estimated. A comparison of our predictions with those given by other references is shown in Tables 3-I–3-III. Hereafter, the masses used as input for our calculation are shown in boldface.

[1] $M_K = (M_{K^0} + M_{K^\pm})/2$, $M_D = (M_{D^0} + M_{D^\pm})/2$, $M_B = (M_{B^0} + M_{B^\pm})/2$. Here and below, all the masses used as input for our calculation are taken from PDG2002 [2].

**Table 1.** The assignment for the meson states lying on different linear Regge trajectories

| Trajectories | Meson states | | | |
|---|---|---|---|---|
| $1\,^1S_0$ | $1\,^1S_0$ $(0^{-+})$, | $1\,^1D_2$ $(2^{-+})$, | $1\,^1G_4$ $(4^{-+})$, | $\cdots$ |
| $2\,^1S_0$ | $2\,^1S_0$ $(0^{-+})$, | $2\,^1D_2$ $(2^{-+})$, | $2\,^1G_4$ $(4^{-+})$, | $\cdots$ |
| $1\,^3S_1$ | $1\,^3S_1$ $(1^{--})$, | $1\,^3D_3$ $(3^{--})$, | $1\,^3G_5$ $(5^{--})$, | $\cdots$ |
| $2\,^3S_1$ | $2\,^3S_1$ $(1^{--})$, | $2\,^3D_3$ $(3^{--})$, | $2\,^3G_5$ $(5^{--})$, | $\cdots$ |
| $1\,^3P_0$ | $1\,^3P_0$ $(0^{++})$, | $1\,^3F_2$ $(2^{++})$, | $1\,^3H_4$ $(4^{++})$, | $\cdots$ |
| $1\,^1P_1$ | $1\,^1P_1$ $(1^{+-})$, | $1\,^1F_3$ $(3^{+-})$, | $1\,^1H_5$ $(5^{+-})$, | $\cdots$ |
| $1\,^3P_1$ | $1\,^3P_1$ $(1^{++})$, | $1\,^3F_3$ $(3^{++})$, | $1\,^3H_5$ $(5^{++})$, | $\cdots$ |
| $1\,^3P_2$ | $1\,^3P_2$ $(2^{++})$, | $1\,^3F_4$ $(4^{++})$, | $1\,^3H_6$ $(6^{++})$, | $\cdots$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\cdots$ |

### 2.2 The $1\,^1P_1$ and $1\,^3P_1$ trajectories

In our calculation of the masses of the states lying on the $1\,^1P_1$ ($1\,^3P_1$) trajectories, we adopt the assumption presented by [7] that the slopes of the parity partners' trajectories coincide, and further, that the slopes do not depend on charge conjugation in accordance with the $C$-invariance of QCD. Under this assumption, the corresponding slopes of the $1\,^1P_1$ ($1\,^3P_1$) are the same as those of the $1\,^1S_0$

**Table 2.** Parameters of the $1\,^1S_0$ trajectories of the form (1)

| | $n\bar{n}$ | $s\bar{s}$ | $n\bar{s}$ | $c\bar{c}$ | $c\bar{n}$ |
|---|---|---|---|---|---|
| $\alpha(0)$ | −0.01316 | −0.3260 | −0.1696 | −3.6178 | −1.8155 |
| $\alpha'$ (GeV$^{-2}$) | 0.7218 | 0.6613 | 0.6902 | 0.4075 | 0.5209 |
| | $c\bar{s}$ | $b\bar{b}$ | $n\bar{b}$ | $s\bar{b}$ | $c\bar{b}$ |
| $\alpha(0)$ | −1.9719 | −17.9790 | −8.9960 | −9.1525 | −10.7980 |
| $\alpha'$ (GeV$^{-2}$) | 0.5043 | 0.2079 | 0.3228 | 0.3164 | 0.2753 |

**Table 3-I.** The masses of the $n\bar{n}$, $s\bar{s}$ and $n\bar{s}$ states lying on the $1\,^1S_0$ trajectories

| | $M_{n\bar{n}}$ (GeV) | | | $M_{s\bar{s}}$ (GeV) | | | $M_{n\bar{s}}$ (GeV) | | |
|---|---|---|---|---|---|---|---|---|---|
| Reference | $J=0$ | $J=2$ | $J=4$ | $J=0$ | $J=2$ | $J=4$ | $J=0$ | $J=2$ | $J=4$ |
| Present work | **0.135** | **1.67** | 2.358 | 0.702 | 1.875 | 2.558 | **0.4957** | **1.773** | 2.458 |
| Exp. [2] | 0.135 | 1.67 | | | | | 0.4957 | 1.773 | |
| [7] | 0.135 | 1.677 | 2.237 | 0.689 | 1.869 | 2.429 | 0.493 | 1.773 | 2.333 |
| [19] | 0.15 | 1.68 | 2.33 | 0.96 | 1.89 | 2.51 | 0.47 | 1.78 | 2.41 |

**Table 3-II.** The masses of the $c\bar{n}$, $c\bar{s}$, $n\bar{b}$ and $s\bar{b}$ states lying on the $1\,^1S_0$ trajectories

| | $M_{c\bar{n}}$ (GeV) | | | $M_{c\bar{s}}$ (GeV) | | | $M_{n\bar{b}}$ (GeV) | | | $M_{s\bar{b}}$ (GeV) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Reference | $J=0$ | $J=2$ | $J=4$ | $J=0$ | $J=2$ | $J=4$ | $J=0$ | $J=2$ | $J=4$ | $J=0$ | $J=2$ | $J=4$ |
| Present work | **1.8669** | 2.706 | 3.341 | 1.977 | 2.806 | 3.441 | **5.2792** | 5.837 | 6.345 | 5.378 | 5.937 | 6.447 |
| Exp. [2] | 1.8669 | | | 1.9685 | | | 5.2792 | | | 5.3696 | | |
| [7] | 1.8641 | 2.692 | 3.228 | 1.971 | 2.786 | 3.323 | 5.2798 | 5.83 | 6.286 | 5.3696 | 5.92 | 6.376 |
| [19] | 1.88 | | | 1.98 | | | 5.31 | | | 5.39 | | |
| [20] | 1.875 | | | 1.981 | | | 5.285 | | | 5.375 | | |
| [21] | 1.865 | | | 1.969 | | | 5.279 | | | 5.383 | | |
| [22] | 1.868 | 2.775 | | 1.965 | 2.900 | | 5.279 | 5.925 | | 5.373 | 6.095 | |
| [23] | 1.85 | 2.74 | 3.24 | 1.94 | 2.86 | 3.37 | 5.28 | 5.96 | 6.36 | 5.37 | 6.07 | 6.47 |

**Table 3-III.** The masses of the $c\bar{c}$, $b\bar{b}$ and $c\bar{b}$ states lying on the $1\,^1S_0$ trajectories

| | $M_{c\bar{c}}$ (GeV) | | | $M_{b\bar{b}}$ (GeV) | | | $M_{c\bar{b}}$ (GeV) | | |
|---|---|---|---|---|---|---|---|---|---|
| Reference | $J=0$ | $J=2$ | $J=4$ | $J=0$ | $J=2$ | $J=4$ | $J=0$ | $J=2$ | $J=4$ |
| Present work | **2.9797** | 3.713 | 4.324 | **9.3** | 9.803 | 10.282 | 6.263 | 6.818 | 7.332 |
| Exp. [2] | 2.9797 | | | 9.3 | | | 6.4 | | |
| [7] | 2.9798 | 3.692 | 4.217 | 9.424 | 9.914 | 10.353 | 6.283 | 6.826 | 7.287 |
| [19] | 2.97 | 3.84 | | 9.40 | 10.15 | | 6.27 | 7.038 | |
| [23] | 3.00 | 3.82 | | 9.41 | 10.15 | | 6.26 | 7.02 | 7.43 |
| [24] | 2.979 | 3.811 | | 9.4 | 10.158 | | 6.270 | 7.078 | |
| [25] | 2.993 | 3.778 | | 9.435 | 10.144 | | 6.313 | 7.027 | |
| [26] | 2.98 | | | 9.377 | 10.127 | | 6.264 | 7.009 | |
| [27] | 2.98 | | | 9.403 | 10.155 | | 6.291 | 7.040 | |
| [28] | 2.98 | | | 9.406 | | | 6.286 | 7.028 | |
| [29] | 2.979 | 3.796 | | 9.359 | 10.142 | | 6.287 | | |
| [30] | 2.979 | | | 9.408 | | | 6.247 | | |
| [31] | 2.921 | 3.867 | | 9.369 | 10.169 | | | | |
| [32] | 2.987 | 3.872 | | 9.413 | 10.167 | | | | |
| [33] | | | | | | | 6.253 | 7.011 | |
| [34] | | | | | | | 6.310 | | |
| [35] | | | | | | | 6.255 | | |
| [36] | | | | | | | 6.280 | | |
| [37] | | | | | | | 6.255 | | |
| [38] | | | | | | | 6.326 | | |
| [39] | | | | | | | 6.386 | | |
| [40] | | | | | | | 6.194–6.292 | | |
| [41] | | | | | | | $\geq 6.220$ | | |

**Table 4.** Parameters of the $1\,{}^1P_1$ trajectories of the form (1)

| | $n\bar{n}$ | $s\bar{s}$ | $n\bar{s}$ | $c\bar{c}$ | $c\bar{n}$ | $c\bar{s}$ |
|---|---|---|---|---|---|---|
| $\alpha(0)$ | −0.0911 | −0.4619 | −0.2765 | −4.0213 | −2.0562 | −2.2416 |
| $\alpha'$ ($\mathrm{GeV}^{-2}$) | 0.7218 | 0.6613 | 0.6902 | 0.4075 | 0.5209 | 0.5043 |

trajectories. Therefore, for the $1\,{}^1P_1$ trajectories, inserting the masses of $b_1(1235)$, $D_1(2420)$ and $D_{s1}(2536)$ as well as the values of $\alpha'_{n\bar{n}}$ , $\alpha'_{c\bar{n}}$ and $\alpha'_{c\bar{s}}$ shown in Table 2 into the following equations:

$$1 = \alpha_{n\bar{n}}(0) + \alpha'_{n\bar{n}} M^2_{b_1(1235)}, \tag{13}$$

$$1 = \alpha_{c\bar{n}}(0) + \alpha'_{c\bar{n}} M^2_{D_1(2420)}, \tag{14}$$

$$1 = \alpha_{c\bar{s}}(0) + \alpha'_{c\bar{s}} M^2_{D_{s1}(2536)}, \tag{15}$$

together with the relations (2) and (3), one can extract the parameters of the $1\,{}^1P_1$ trajectories summarized in Table 4. With these parameters, the masses of the $J = 1$, $J = 3$ and $J = 5$ states lying on the $1\,{}^1P_1$ trajectories can be obtained. Comparison of our calculations with those performed by other references is given in Tables 5-I–5-II.

For the $1\,{}^3P_1$ trajectories, in order to extract the value of $\alpha_{n\bar{s}}(0)$, we should know the mass of $K_{1A}$ since the states containing the $s$-quarks of the $1\,{}^3P_1$ multiplet are not established experimentally except for $K_{1A}$. It is well-known that the physical states $K_1(1400)$ and $K_1(1270)$ are mixtures of $K_{1A}$ and $K_{1B}$, the strange members of $1\,{}^3P_1$ and $1\,{}^1P_1$; therefore, the masses of $K_{1A}$, $K_{1B}$, $K_1(1270)$ and $K_1(1400)$ must obey the relation [42]

$$M^2_{K_{1A}} + M^2_{K_{1B}} = M^2_{K_1(1400)} + M^2_{K_1(1270)}; \tag{16}$$

then, using $M_{K_1(1400)} = 1.402\,\mathrm{GeV}$, $M_{K_1(1270)} = 1.273\,\mathrm{GeV}$ [2], and $M_{K_{1B}} = 1.36\,\mathrm{GeV}$ shown in Table 5-I, one can have $M_{K_{1A}} = 1.318\,\mathrm{GeV}$. Based on the values of the slopes shown in Table 2, inserting $M_{K_{1A}} = 1.318\,\mathrm{GeV}$ as well as the masses of $a_1(1260)$, $\chi_{c1}(1P)$ and $\chi_{b1}(1P)$ into the following equations:

$$1 = \alpha_{n\bar{n}}(0) + \alpha'_{n\bar{n}} M^2_{a_1(1260)}, \tag{17}$$

$$1 = \alpha_{n\bar{s}}(0) + \alpha'_{n\bar{s}} M^2_{K_{1A}}, \tag{18}$$

$$1 = \alpha_{c\bar{c}}(0) + \alpha'_{c\bar{c}} M^2_{\chi_{c1}(1P)}, \tag{19}$$

$$1 = \alpha_{b\bar{b}}(0) + \alpha'_{b\bar{b}} M^2_{\chi_{b1}(1P)}, \tag{20}$$

**Table 5-I.** The masses of the $n\bar{n}$, $s\bar{s}$ and $n\bar{s}$ states lying on the $1\,{}^1P_1$ trajectories

| | $M_{n\bar{n}}$ (GeV) | | | $M_{s\bar{s}}$ (GeV) | | | $M_{n\bar{s}}$ (GeV) | | |
|---|---|---|---|---|---|---|---|---|---|
| Reference | $J = 1$ | $J = 3$ | $J = 5$ | $J = 1$ | $J = 3$ | $J = 5$ | $J = 1$ | $J = 3$ | $J = 5$ |
| Present work | **1.2295** | 2.069 | 2.656 | 1.487 | 2.288 | 2.874 | 1.360 | 2.179 | 2.765 |
| Exp. [2] | 1.2295 | | | | | | | | |
| [7] | | | | | | | | | |
| [19] | 1.22 | 2.03 | | 1.47 | 2.22 | | 1.34 | 2.12 | |

**Table 5-II.** The masses of the $c\bar{n}$, $c\bar{s}$ and $c\bar{c}$ states lying on the $1\,{}^1P_1$ trajectories

| | $M_{c\bar{n}}$ (GeV) | | | $M_{c\bar{s}}$ (GeV) | | | $M_{c\bar{c}}$ (GeV) | | |
|---|---|---|---|---|---|---|---|---|---|
| Reference | $J = 1$ | $J = 3$ | $J = 5$ | $J = 1$ | $J = 3$ | $J = 5$ | $J = 1$ | $J = 3$ | $J = 5$ |
| Present work | **2.4222** | 3.116 | 3.681 | **2.53535** | 3.224 | 3.789 | 3.510 | 4.151 | 4.705 |
| Exp. [2] | 2.4222 | | | 2.53535 | | | 3.526 | | |
| [19] | 2.44 | | | 2.53 | | | 3.52 | 4.09 | |
| [20] | 2.414 | | | 2.515 | | | | | |
| [21] | 2.421 | | | 2.537 | | | | | |
| [22] | 2.417 | 3.074 | | 2.535 | 3.203 | | | | |
| [23] | 2.40 | 3.01 | | 2.51 | 3.13 | | 3.51 | 4.06 | |
| [24] | | | | | | | 3.526 | | |
| [25] | | | | | | | 3.458 | | |
| [26] | | | | | | | 3.493 | | |
| [27] | | | | | | | 3.521 | | |
| [28] | | | | | | | 3.501 | | |
| [29] | | | | | | | 3.496 | | |
| [30] | | | | | | | 3.526 | | |
| [31] | | | | | | | 3.525 | | |
| [32] | | | | | | | 3.529 | | |

**Table 6.** Parameters of the $1\,^3P_1$ trajectories of the form (1)

| | $n\bar{n}$ | $s\bar{s}$ | $n\bar{s}$ | $c\bar{c}$ | $c\bar{n}$ |
|---|---|---|---|---|---|
| $\alpha(0)$ | −0.0920 | −0.3060 | −0.1990 | −4.0219 | −2.0570 |
| $\alpha'$ (GeV$^{-2}$) | 0.7218 | 0.6613 | 0.6902 | 0.4075 | 0.5209 |
| | $c\bar{s}$ | $b\bar{b}$ | $n\bar{b}$ | $s\bar{b}$ | $c\bar{b}$ |
| $\alpha(0)$ | −2.1640 | −19.3462 | −9.7191 | −9.8261 | −11.6841 |
| $\alpha'$ (GeV$^{-2}$) | 0.5043 | 0.2079 | 0.3228 | 0.3164 | 0.2753 |

and using the relations (2) and (3), one can extract the parameters of the $1\,^3P_1$ trajectories summarized in Table 6. In terms of these parameters, the masses of the $J=1$, $J=3$ and $J=5$ states lying on the $1\,^3P_1$ trajectories can be given. Comparison of our calculations with those performed by other references is shown in Tables 7-I–7-III.

### 2.3 The $1\,^3S_1$ trajectories

Inserting the masses of $\rho$, $\rho_3(1690)$, $K^*(892)$, $K_3^*(1780)$, $J/\psi$, $D^*(2010)$[2], $\Upsilon$ and $B^*$ into the following equations:

$$1=\alpha_{n\bar{n}}(0)+\alpha'_{n\bar{n}}M^2_{\rho}, \tag{21}$$

$$3=\alpha_{n\bar{n}}(0)+\alpha'_{n\bar{n}}M^2_{\rho_3(1690)}, \tag{22}$$

$$1=\alpha_{n\bar{s}}(0)+\alpha'_{n\bar{s}}M^2_{K^*(892)}, \tag{23}$$

$$3=\alpha_{n\bar{s}}(0)+\alpha'_{n\bar{s}}M^2_{K_3^*(1780)}, \tag{24}$$

$$1=\alpha_{c\bar{c}}(0)+\alpha'_{c\bar{c}}M^2_{J/\psi}, \tag{25}$$

$$1=\alpha_{c\bar{n}}(0)+\alpha'_{c\bar{n}}M^2_{D^*(2010)}, \tag{26}$$

$$1=\alpha_{b\bar{b}}(0)+\alpha'_{b\bar{b}}M^2_{\Upsilon(1S)}, \tag{27}$$

$$1=\alpha_{n\bar{b}}(0)+\alpha'_{n\bar{b}}M^2_{B^*}, \tag{28}$$

and by means of the relations (2) and (3), one can extract the parameters of the $1\,^3S_1$ trajectories as shown in Table 8. For the masses of the $J=1$, $J=3$ and $J=5$ states lying on the $1\,^3S_1$ trajectories, our calculations and those performed by other references are shown in Tables 9-I–9-III.

### 2.4 The $1\,^3P_2$ trajectories

Because the tensor meson trajectories are the parity partners of the vector meson trajectories, according to the assumption mentioned in Sect. 2.2, the corresponding slopes of the tensor meson trajectories are the same as those of the vector meson trajectories. So, according to the values of the slopes shown in Table 8, in the presence of the relations (2) and (3), inserting the masses of $a_2(1320)$, $K_2^*(1430)$[3], $\chi_{c2}(1P)$ and $\chi_{b2}(1P)$ into the following equations:

$$2=\alpha_{n\bar{n}}(0)+\alpha'_{n\bar{n}}M^2_{a_2(1320)}, \tag{29}$$

$$2=\alpha_{n\bar{s}}(0)+\alpha'_{n\bar{s}}M^2_{K_2^*(1430)}, \tag{30}$$

$$2=\alpha_{c\bar{c}}(0)+\alpha'_{c\bar{c}}M^2_{\chi_{c2}(1P)}, \tag{31}$$

$$2=\alpha_{b\bar{b}}(0)+\alpha'_{b\bar{b}}M^2_{\chi_{b2}(1P)}, \tag{32}$$

one can extract the parameters of the tensor meson trajectories as summarized in Table 10. Based on these parameters, the masses of the $J=2$, $J=4$ and $J=6$ states lying on the tensor meson trajectories can be obtained. Comparison of our predictions with those given by other references is shown in Tables 11-I–11-III.

### 2.5 The $2\,^3S_1$ trajectories

Finally, we wish to discuss the $2\,^3S_1$ trajectories. According to [5], the corresponding slopes of the $2\,^3S_1$ and $1\,^3S_1$ trajectories coincide. Therefore, based on the values of the slopes shown in Table 8, inserting the masses of $\rho(1450)$, $\psi(2S)$ and $\Upsilon(2S)$ into the following equations:

$$1=\alpha_{n\bar{n}}(0)+\alpha'_{n\bar{n}}M^2_{\rho(1450)}, \tag{33}$$

$$1=\alpha_{c\bar{c}}(0)+\alpha'_{c\bar{c}}M^2_{\psi(2S)}, \tag{34}$$

$$1=\alpha_{b\bar{b}}(0)+\alpha'_{b\bar{b}}M^2_{\Upsilon(2S)}, \tag{35}$$

one can extract the values of $\alpha_{n\bar{n}}(0)$, $\alpha_{c\bar{c}}(0)$ and $\alpha_{b\bar{b}}(0)$. In order to obtain the parameters of the trajectories on which the states containing $s$-quarks lie, we should have the masses of the states containing $s$-quarks of the $2\,^3S_1$ multiplet. According to [2], only the mass of the state $\phi(1680)$ is well established experimentally[4]. Also, the physical state $\phi(1680)$ is usually believed to be a mixture of $s\bar{s}$ with a light quark–antiquark component. Therefore, we are forced to make an additional assumption. In our calculation, we assume that the masses of the $\phi(1680)$ and the pure $s\bar{s}$ state of the $2\,^3S_1$ multiplet are close. As we know, $M_\rho \simeq M_\omega$ implies that the $\omega$ and $\phi$ are almost ideally mixing [43], that is to say, the masses of the physical state the $\phi$ and the pure $s\bar{s}$ state of the $1\,^3S_1$ multiplet are close. In fact, comparison of our prediction shown in Table 9-I, $M_{s\bar{s}}=1.01\,\text{GeV}$, with the mass of the physical state $\phi$, $M_\phi=1.019456\,\text{GeV}$ [2], already clearly indicates that the masses of $\phi$ and the pure $s\bar{s}$ state of the $1\,^3S_1$ multiplet are close. Based on the fact that $M_{\rho(1450)}\simeq M_{\omega(1420)}$ [2], in analogy with $\omega$ versus $\phi$, the physical states $\omega(1420)$ and $\phi(1680)$ can be considered to be almost ideally mixing. In addition, the decay modes of the $\phi(1680)$ [2] strongly imply that the physical state $\phi(1680)$ is almost a pure $s\bar{s}$ state. We therefore conclude that our assumption that the masses of the $\phi(1680)$ and the pure $s\bar{s}$ state of the $2\,^3S_1$ multiplet are close is plausible. Under this assumption, the $\alpha_{s\bar{s}}(0)$ can be obtained from $1=\alpha_{s\bar{s}}(0)+\alpha'_{s\bar{s}}M^2_{\phi(1680)}$.

Based on the values of $\alpha_{n\bar{n}}(0)$, $\alpha_{c\bar{c}}(0)$, $\alpha_{b\bar{b}}(0)$ and $\alpha_{s\bar{s}}(0)$, from the relations (2) and (3), all the parameters of the $2\,^3S_1$ trajectories are presented in Table 12. The masses of the $J=1$, $J=3$ and $J=5$ states lying on the $2\,^3S_1$ trajectories given by the present work as well as those predicted by other references are shown in Tables 13-I–13-III.

[2] $M_{K^*(892)}=(M_{K^*(892)^\pm}+M_{K^*(892)^0})/2$, $M_{D^*(2010)}=(M_{D^*(2010)^\pm}+M_{K^*(2007)^0})/2$.

[3] $M_{K_2^*(1430)}=(M_{K_2^*(1430)^\pm}+M_{K_2^*(1430)^0})/2$.

[4] The assignment that $K^*(1410)$ belongs to the $2\,^3S_1$ multiplet is problematic, which will be discussed below.

**Table 7-I.** The masses of the $n\bar{n}$, $s\bar{s}$ and $n\bar{s}$ states lying on the $1\,^3P_1$ trajectories

| | $M_{n\bar{n}}$ (GeV) | | | $M_{s\bar{s}}$ (GeV) | | | $M_{n\bar{s}}$ (GeV) | | |
|---|---|---|---|---|---|---|---|---|---|
| Reference | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ |
| Present work | **1.23** | 2.070 | 2.656 | 1.405 | 2.236 | 2.833 | **1.318** | 2.153 | 2.745 |
| Exp. [2] | 1.23 | | | | | | | | |
| [7] | 1.230 | 2.000 | 2.427 | 1.501 | 2.218 | 2.643 | 1.368 | 2.109 | 2.535 |
| [19] | 1.24 | 2.05 | | 1.48 | 2.23 | | 1.38 | 2.15 | |

**Table 7-II.** The masses of the $c\bar{n}$, $c\bar{s}$, $n\bar{b}$ and $s\bar{b}$ states lying on the $1\,^3P_1$ trajectories

| | $M_{c\bar{n}}$ (GeV) | | | $M_{c\bar{s}}$ (GeV) | | | $M_{n\bar{b}}$ (GeV) | | | $M_{s\bar{b}}$ (GeV) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Reference | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ |
| Present work | 2.423 | 3.116 | 3.681 | 2.505 | 3.200 | 3.769 | 5.763 | 6.277 | 6.753 | 5.849 | 6.367 | 6.845 |
| Exp. [2] | | | | | | | | | | | | |
| [7] | 2.418 | 3.042 | 3.473 | 2.5354 | 3.15 | 3.58 | 5.692 | 6.171 | 6.57 | 5.796 | 6.273 | 6.671 |
| [19] | 2.49 | | | 2.57 | | | | | | | | |
| [20] | 2.501 | | | 2.569 | | | 5.757 | | | 5.859 | | |
| [21] | 2.407 | | | 2.521 | | | 5.731 | | | 5.855 | | |
| [22] | 2.49 | 3.123 | | 2.605 | 3.247 | | 5.742 | 6.271 | | 5.842 | 6.376 | |
| [23] | 2.41 | 3.03 | | 2.52 | 3.15 | | 5.69 | 6.20 | | 5.80 | 6.31 | |

**Table 7-III.** The masses of the $c\bar{c}$, $b\bar{b}$ and $c\bar{b}$ states lying on the $1\,^3P_1$ trajectories

| | $M_{c\bar{c}}$ (GeV) | | | $M_{b\bar{b}}$ (GeV) | | | $M_{c\bar{b}}$ (GeV) | | |
|---|---|---|---|---|---|---|---|---|---|
| Reference | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ |
| Present work | **3.51051** | 4.151 | 4.705 | **9.8927** | 10.368 | 10.822 | 6.788 | 7.303 | 7.785 |
| Exp. [2] | 3.51051 | | | 9.8927 | | | | | |
| [7] | 3.5105 | 4.08 | 4.513 | 9.8919 | 10.33 | 10.727 | 6.74 | 7.215 | 7.62 |
| [19] | 3.51 | 4.10 | | 9.88 | 10.35 | | 6.742 | | |
| [23] | 3.50 | 4.06 | | 9.87 | 10.36 | | 6.74 | 7.25 | |
| [24] | 3.510 | | | 9.892 | | | 6.736 | | |
| [25] | 3.435 | | | 9.885 | | | 6.721 | | |
| [26] | 3.486 | | | 9.864 | | | 6.730 | | |
| [27] | 3.502 | | | 9.891 | 10.347 | | 6.754 | | |
| [28] | 3.482 | | | 9.891 | | | 6.737 | | |
| [29] | 3.482 | | | 9.895 | | | | | |
| [30] | 3.511 | | | 9.893 | | | 6.742 | | |
| [31] | 3.506 | | | 9.893 | | | | | |
| [32] | 3.513 | | | 9.893 | | | | | |
| [33] | | | | | | | 6.718 | | |
| [34] | | | | | | | 6.760 | | |
| [35] | | | | | | | 6.730 | | |
| [36] | | | | | | | 6.750 | | |

## 3 Discussions of our results

Comparison of the predictions given by the present work with those given by other references clearly illustrates that the agreement between our results and those of many other approaches is generally satisfactory, which indicates that in the presence of the relations (2) and (3), the quasi-linear Regge trajectory can give a reasonable description for the spectrum of both light and heavy mesons.

From the parameters shown in Tables 2 and 8, we find that the slopes of Regge trajectories are flavor-dependent and approximately satisfy $a'_{n\bar{n}} > a'_{n\bar{s}} > a'_{s\bar{s}} > a'_{c\bar{n}} > a'_{c\bar{s}} > a'_{c\bar{c}} > a'_{n\bar{b}} > a'_{s\bar{b}} > a'_{c\bar{b}} > a'_{b\bar{b}}$. Tables 2 and 8 also indicate that for the trajectories on which the states containing $b$-quarks lie, the slopes of the $1\,^1S_0$ trajectories approximately are the same as those of the $1\,^3S_1$ trajectories.

**Table 8.** Parameters of the $1\,^3S_1$ trajectories of the form (1)

| | $n\bar{n}$ | $s\bar{s}$ | $n\bar{s}$ | $c\bar{c}$ | $c\bar{n}$ |
|---|---|---|---|---|---|
| $\alpha(0)$ | 0.4749 | 0.1675 | 0.3212 | −3.1851 | −1.3551 |
| $\alpha'$ (GeV$^{-2}$) | 0.8830 | 0.8181 | 0.8493 | 0.4364 | 0.5841 |
| | $c\bar{s}$ | $b\bar{b}$ | $n\bar{b}$ | $s\bar{b}$ | $c\bar{b}$ |
| $\alpha(0)$ | −1.5088 | −17.338 | −8.4316 | −8.5853 | −10.2616 |
| $\alpha'$ (GeV$^{-2}$) | 0.5692 | 0.2049 | 0.3326 | 0.3277 | 0.2789 |

From the relations (1) and (2), the following quadratic mass relation can be obtained:

$$\alpha'_{i\bar{i}}M^2_{i\bar{i}} + \alpha'_{j\bar{j}}M^2_{j\bar{j}} = 2\alpha'_{j\bar{i}}M^2_{j\bar{i}}. \tag{36}$$

With the help of the relation (36) and the parameters shown in Tables 2 and 8, one can have

$$\left.\begin{array}{l} 8.13M^2_{n\bar{s}} + 4.80M^2_{c\bar{c}} = 6.14M^2_{c\bar{n}} + 5.94M^2_{c\bar{s}} \\ 17.1M^2_{n\bar{s}} + 5.15M^2_{b\bar{b}} = 8.0M^2_{n\bar{b}} + 7.84M^2_{s\bar{b}} \end{array}\right\} \text{for the } 1\,^1S_0\text{-like trajectories}, \tag{37}$$

$$\left.\begin{array}{l} 8.13M^2_{n\bar{s}} + 4.18M^2_{c\bar{c}} = 5.59M^2_{c\bar{n}} + 5.45M^2_{c\bar{s}} \\ 17.1M^2_{n\bar{s}} + 4.13M^2_{b\bar{b}} = 6.70M^2_{n\bar{b}} + 6.60M^2_{s\bar{b}} \end{array}\right\} \text{for the } 1\,^3S_1\text{-like trajectories}; \tag{38}$$

here and below, the $1\,^1S_0$-like trajectories ($1\,^3S_1$-like trajectories) denote the trajectories whose slopes coincide with those of the $1\,^1S_0$ trajectories ($1\,^3S_1$ trajectories). The similar relations

$$\left.\begin{array}{l} 8.13M^2_{n\bar{s}} + 4.75M^2_{c\bar{c}} = 6M^2_{c\bar{n}} + 6M^2_{c\bar{s}} \\ 17.1M^2_{n\bar{s}} + 3.64M^2_{b\bar{b}} = 6M^2_{n\bar{b}} + 6M^2_{s\bar{b}} \end{array}\right\} \text{for all the trajectories} \tag{39}$$

have been proposed in [8] based on the simplification that the Regge slopes in the light quark sector are the same for all the meson multiplets. The non-integer coefficients in (37), (38) and (39) reflect the uncertainty in fitting the values of the Regge slopes. Note that the accuracy of our results, (37) and (38), is better than that of (39). For example, for the $1\,^3S_1$ multiplet, $17.1M^2_{n\bar{s}} + 4.13M^2_{b\bar{b}} = 6.70M^2_{n\bar{b}} + 6.60M^2_{s\bar{b}}$ gives 383.29 GeV$^2$ on the LHS versus 383.58 GeV$^2$ on the RHS, with an accuracy of $\sim 0.08\%$, while $17.1M^2_{n\bar{s}} + 3.64M^2_{b\bar{b}} = 6M^2_{n\bar{b}} + 6M^2_{s\bar{b}}$ gives 339.44 GeV$^2$ on the LHS versus 346.13 GeV$^2$ on the RHS, with an accuracy of $\sim 2\%$.

Also, based on the predicted masses shown in Sect. 2, and comparing $M^2_{n\bar{n}} + M^2_{s\bar{s}}$ with $2M^2_{n\bar{s}}$, one can find that the relation $M^2_{n\bar{n}} + M^2_{s\bar{s}} = 2M^2_{n\bar{s}}$ holds with an accuracy of $\sim 4\%$ for the $1\,^1S_0$ multiplet, and with an accuracy of $\leq 1\%$ for the remaining multiplets considered in our present work. In fact, from the parameters shown in Tables 2 and 8, together with the formula (36), we can have

$$\begin{array}{l} 1.09M^2_{n\bar{n}} + M^2_{s\bar{s}} = 2\times 1.04M^2_{n\bar{s}} \\ \quad \text{for } 1\,^1S_0\text{-like trajectories}, \\ 1.08M^2_{n\bar{n}} + M^2_{s\bar{s}} = 2\times 1.04M^2_{n\bar{s}} \\ \quad \text{for } 1\,^3S_1\text{-like trajectories}, \end{array} \tag{40}$$

from which one can naturally expect that the Gell-Mann–Okubo mass relation $M^2_{n\bar{n}} + M^2_{s\bar{s}} = 2M^2_{n\bar{s}}$ [44] can hold for all the multiplets with a good accuracy.

In our present work, the masses of the strange members of $1\,^1P_1$ and $1\,^3P_1$, $M_{K_{1B}}$ and $M_{K_{1A}}$, are determined to have the values of 1.36 GeV and 1.318 GeV, respectively. Inserting $M_{K_{1B}}$ and $M_{K_{1A}}$ into the relations [42]

$$\tan^2(2\theta_K) = \left[\frac{M^2_{K_1(1400)} - M^2_{K_1(1270)}}{M^2_{K_{1A}} - M^2_{K_{1B}}}\right]^2 - 1, \tag{41}$$

**Table 9-I.** The masses of the $n\bar{n}$, $s\bar{s}$ and $n\bar{s}$ states lying on the $1\,^3S_1$ trajectories

| | $M_{n\bar{n}}$ | | | $M_{s\bar{s}}$ | | | $M_{n\bar{s}}$ | | |
|---|---|---|---|---|---|---|---|---|---|
| Reference | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ |
| Present work | **0.7711** | **1.691** | 2.264 | 1.01 | 1.861 | 2.430 | **0.894** | 1.776 | 2.347 |
| Exp. [2] | 0.7711 | 1.691 | | | | | 0.894 | 1.776 | |
| [7] | 0.769 | 1.6888 | 2.124 | 1.015 | 1.863 | 2.215 | 0.8961 | 1.776 | 2.215 |
| [19] | 0.77 | 1.68 | 2.30 | 1.02 | 1.90 | 2.47 | 0.90 | 1.79 | 2.39 |

**Table 9-II.** The masses of the $c\bar{n}$, $c\bar{s}$, $n\bar{b}$ and $s\bar{b}$ states lying on the $1\,^3S_1$ trajectories

| | $M_{c\bar{n}}$ | | | $M_{c\bar{s}}$ | | | $M_{n\bar{b}}$ | | | $M_{s\bar{b}}$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Reference | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ |
| Present work | **2.008** | 2.731 | 3.299 | 2.10 | 2.815 | 3.382 | **5.325** | 5.863 | 6.355 | 5.408 | 5.946 | 6.438 |
| Exp. [2] | 2.008 | | | 2.1124 | | | 5.325 | | | 5.4166 | | |
| [7] | 2.0067 | 2.721 | 3.191 | 2.102 | 2.808 | 3.279 | 5.3249 | 5.814 | 6.217 | 5.411 | 5.901 | 6.306 |
| [19] | 2.04 | 2.83 | | 2.13 | 2.92 | | 5.37 | 6.11 | | 5.45 | 6.18 | |
| [20] | 2.009 | | | 2.111 | | | 5.324 | | | 5.412 | | |
| [21] | 2.007 | | | 2.111 | | | 5.324 | | | 5.432 | | |
| [22] | 2.005 | 2.799 | | 2.113 | 2.925 | | 5.324 | 6.025 | | 5.421 | 6.127 | |
| [23] | 2.02 | 2.78 | | 2.13 | 2.90 | | 5.33 | 5.97 | | 5.43 | 6.08 | |

**Table 9-III.** The masses of the $c\bar{c}$, $b\bar{b}$ and $c\bar{b}$ states lying on the $1\,^3S_1$ trajectories

| | $M_{c\bar{c}}$ | | | $M_{b\bar{b}}$ | | | $M_{c\bar{b}}$ | | |
|---|---|---|---|---|---|---|---|---|---|
| Reference | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ |
| Present work | **3.09687** | 3.765 | 4.331 | **9.4603** | 9.963 | 10.44 | 6.354 | 6.896 | 7.397 |
| Exp. [2] | 3.09687 | | | 9.4603 | | | | | |
| [7] | 3.0969 | 3.753 | 4.24 | 9.4604 | 9.906 | 10.30 | 6.356 | 6.853 | 7.276 |
| [19] | 3.1 | 3.85 | | 9.46 | 10.16 | | 6.34 | 7.04 | |
| [23] | 3.10 | 3.83 | | 9.46 | 10.15 | | 6.34 | 7.04 | |
| [24] | 3.096 | 3.815 | | 9.460 | 10.162 | | 6.332 | 7.081 | |
| [25] | 3.093 | 3.913 | | 9.451 | 10.165 | | 6.347 | 7.086 | |
| [26] | 3.097 | | | 9.464 | 10.130 | | 6.337 | 7.005 | |
| [27] | 3.097 | | | 9.460 | 10.163 | | 6.349 | 7.049 | |
| [28] | 3.098 | | | 9.461 | | | 6.341 | 7.032 | |
| [29] | 3.118 | | | 9.462 | 10.149 | | 6.372 | | |
| [30] | 3.097 | | | 9.460 | | | 6.308 | | |
| [31] | 3.125 | 3.867 | | 9.461 | 10.172 | | | | |
| [32] | 3.104 | 3.884 | | 9.459 | 10.172 | | | | |
| [33] | | | | | | | 6.317 | | |
| [34] | | | | | | | 6.355 | | |
| [35] | | | | | | | 6.320 | | |
| [36] | | | | | | | 6.321 | | |
| [37] | | | | | | | 6.333 | | |
| [40] | | | | | | | 6.284–6.357 | | |
| [41] | | | | | | | $\geq 6.279$ | | |

$$M_{K_{1A}} = \sqrt{M^2_{K_1(1400)}\cos^2\theta_K + M^2_{K_1(1270)}\sin^2\theta_K}, \quad (42)$$

one can extract $\theta_K$, the mixing angle of $K_{1A}$ and $K_{1B}$, is about 54.5°. Such a result is inconsistent with $\theta_K \sim 33°$ suggested in [19,42]; however, it should be noted that the masses of $K_{1B}$ and $K_{1A}$ predicted by us are in excellent agreement with the results that $M_{K_{1B}} = 1356\,\text{MeV}$ and $M_{K_{1A}} = 1322\,\text{MeV}$ suggested by Burakovsky and Goldman in a non-relativistic constituent quark model [45], and also with the results that $M_{K_{1B}} \simeq 1.368\,\text{MeV}$ and $M_{K_{1A}} \simeq 1.31\,\text{GeV}$ given in [46]. Further, taking as input $M_{s\bar{s}} = 1.405\,\text{GeV}$ which is obtained in the presence of $M_{K_{1A}} = 1.318\,\text{GeV}$, and repeating the calculation of our previous paper [47], one can have $|f_1(1410)\rangle = \cos\theta|8\rangle - \sin\theta|1\rangle$ and $|f_1(1285)\rangle = \sin\theta|8\rangle + \cos\theta|1\rangle$ with $\theta = 47.3°$, which is in agreement with the value of $\theta \sim 50°$ suggested by Close and Kirk [48].

The Particle Data Group states that the $K^*(1410)$ could be replaced by the $K^*(1680)$ as the $2\,^3S_1$ state [2]. The problem with the $K^*(1410)$ is that it is much too light to be the $2\,^3S_1$ state, even if one takes into account $2\,^3S_1$–$1\,^3D_1$ mixing; therefore, it is suggested by Törnqvist [49] that one can well doubt the existence of the $K^*(1410)$. In Table 13-I, the mass of the strange member of the $2\,^3S_1$ multiplet is determined to have the value 1.573 GeV, which is in excellent agreement with the value of 1.58 GeV predicted by Godfrey and Isgur [19] in a relativistic quark model. Comparison of 1573 MeV and $1414 \pm 15$ MeV, the mass of the state $K^*(1410)$ [2], would challenge the assignment that the state $K^*(1410)$ is the strange member of the $2\,^3S_1$ multiplet. It has been suggested [49] that the state $K^*(1680)$ should be resolved into two separate states of normal widths ($\Gamma \approx 150\,\text{MeV}$) fitting well the $1\,^3D_1$ ($\approx 1784\,\text{MeV}$) and $2\,^3S_1$ ($\approx 1608\,\text{MeV}$) states. Our results support the state $K^*(1573)$, rather than the $K^*(1410)$, being the member of the $2\,^3S_1$ multiplet, which also agrees with the conclusion given by [50].

The masses of the pure $s\bar{s}$ states predicted by us cannot be directly measured experimentally, since the pure isoscalar $n\bar{n}$ and $s\bar{s}$ states usually can mix. However, comparison of the mass of the pure $s\bar{s}$ state with that of the physical states (mainly SU(3) singlet) can help us to understand the mixing of the two isoscalar physical states of a meson nonet. For example, for the $1\,^1S_0$ nonet, $M_{\eta'} \approx 9.578\,\text{GeV}$ [2] and $M_{s\bar{s}} \approx 0.702\,\text{GeV}$ shown in Table 3-I, which implies that the $\eta$ and $\eta'$ must be non-ideally mixing. However, for the $1\,^3S_1$ nonet, $M_\phi \approx 1.02\,\text{GeV}$ [2] and $M_{s\bar{s}} \approx 1.01\,\text{GeV}$ shown in Table 9-I, which implies that the $\omega$ and $\phi$ are almost ideally mixing. The above deduction is consistent

**Table 10.** Parameters of the $1\,^3P_2$ trajectories of the form (1)

| | $n\bar{n}$ | $s\bar{s}$ | $n\bar{s}$ | $c\bar{c}$ | $c\bar{n}$ |
|---|---|---|---|---|---|
| $\alpha(0)$ | 0.4661 | 0.0653 | 0.2657 | −3.5189 | −1.5264 |
| $\alpha'$ (GeV$^{-2}$) | 0.8830 | 0.8181 | 0.8493 | 0.4364 | 0.5841 |
| | $c\bar{s}$ | $b\bar{b}$ | $n\bar{b}$ | $s\bar{b}$ | $c\bar{b}$ |
| $\alpha(0)$ | −1.7268 | −18.1334 | −8.8337 | −9.0341 | −10.8261 |
| $\alpha'$ (GeV$^{-2}$) | 0.5692 | 0.2049 | 0.3326 | 0.3277 | 0.2789 |

**Table 11-I.** The masses of the $n\bar{n}$, $s\bar{s}$ and $n\bar{s}$ states lying on the 1 $^3P_2$ trajectories

| | $M_{n\bar{n}}$ | | | $M_{s\bar{s}}$ | | | $M_{n\bar{s}}$ | | |
|---|---|---|---|---|---|---|---|---|---|
| Reference | $J=2$ | $J=4$ | $J=6$ | $J=2$ | $J=4$ | $J=6$ | $J=2$ | $J=4$ | $J=6$ |
| Present work | **1.318** | 2.001 | 2.503 | 1.538 | 2.193 | 2.694 | **1.429** | 2.097 | 2.598 |
| Exp. [2] | 1.318 | 2.011 | | | | | 1.429 | 2.045 | |
| [7] | 1.3181 | 1.927 | 2.256 | 1.544 | 2.124 | 2.457 | 1.4324 | 2.026 | 2.357 |
| [19] | 1.31 | 2.01 | | 1.53 | 2.20 | | 1.43 | 2.11 | |

**Table 11-II.** The masses of the $c\bar{n}$, $c\bar{s}$, $n\bar{b}$ and $s\bar{b}$ states lying on the 1 $^3P_2$ trajectories

| | $M_{c\bar{n}}$ | | | $M_{c\bar{s}}$ | | | $M_{n\bar{b}}$ | | | $M_{s\bar{b}}$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Reference | $J=2$ | $J=4$ | $J=6$ | $J=2$ | $J=4$ | $J=6$ | $J=2$ | $J=4$ | $J=6$ | $J=2$ | $J=4$ | $J=6$ |
| Present work | 2.457 | 3.076 | 3.590 | 2.559 | 3.172 | 3.684 | 5.707 | 6.212 | 6.678 | 5.803 | 6.307 | 6.773 |
| Exp. [2] | 2.4589 | | | | | | | | | | | |
| [7] | 2.454 | 3.01 | 3.39 | 2.56 | 3.109 | 3.489 | 5.698 | 6.122 | 6.472 | 5.797 | 6.22 | 6.57 |
| [19] | 2.50 | 3.11 | | 2.59 | 3.19 | | 5.80 | 6.36 | | 5.88 | 6.43 | |
| [20] | 2.459 | | | 2.560 | | | 5.733 | | | 5.844 | | |
| [21] | 2.465 | | | 2.573 | | | 5.759 | | | 5.875 | | |
| [22] | 2.460 | 3.091 | | 2.581 | 3.220 | | 5.714 | 6.226 | | 5.820 | 6.337 | |
| [23] | 2.46 | 3.03 | | 2.58 | 3.16 | | 5.71 | 6.18 | | 5.82 | 6.29 | |

**Table 11-III.** The masses of the $c\bar{c}$, $b\bar{b}$ and $c\bar{b}$ states lying on the 1 $^3P_2$ trajectories

| | $M_{c\bar{c}}$ | | | $M_{b\bar{b}}$ | | | $M_{c\bar{b}}$ | | |
|---|---|---|---|---|---|---|---|---|---|
| Reference | $J=2$ | $J=4$ | $J=6$ | $J=2$ | $J=4$ | $J=6$ | $J=2$ | $J=4$ | $J=6$ |
| Present work | **3.55618** | 4.151 | 4.670 | **9.9126** | 10.393 | 10.853 | 6.781 | 7.291 | 7.767 |
| Exp. [2] | 3.55618 | | | 9.9126 | | | | | |
| [7] | 3.5562 | 4.092 | 4.498 | 9.9132 | 10.31 | 10.665 | 6.78 | 7.213 | 7.582 |
| [19] | 3.55 | 4.06 | | 9.89 | 10.36 | | 6.77 | 7.27 | |
| [23] | 3.54 | 4.09 | | 9.90 | 10.36 | | 6.76 | 7.25 | |
| [24] | 3.556 | | | 9.913 | | | 6.762 | | |
| [25] | 3.589 | | | 9.921 | | | 6.800 | | |
| [26] | 3.507 | | | 9.886 | | | 6.747 | | |
| [27] | 3.556 | | | 9.913 | 10.353 | | 6.787 | | |
| [28] | 3.530 | | | 9.910 | | | 6.772 | | |
| [29] | 3.527 | | | 9.917 | | | | | |
| [30] | 3.557 | | | 9.914 | | | 6.773 | | |
| [31] | 3.561 | | | 9.912 | | | | | |
| [32] | 3.557 | | | 9.911 | | | | | |
| [33] | | | | | | | 6.743 | | |
| [34] | | | | | | | 6.773 | | |
| [35] | | | | | | | 6.770 | | |
| [36] | | | | | | | 6.783 | | |

**Table 12.** Parameters of the 2 $^3S_1$ trajectories of the form (1)

| | $n\bar{n}$ | $s\bar{s}$ | $n\bar{s}$ | $c\bar{c}$ | $c\bar{n}$ |
|---|---|---|---|---|---|
| $\alpha(0)$ | −0.8951 | −1.3090 | −1.1021 | −4.9291 | −2.9121 |
| $\alpha'$ (GeV$^{-2}$) | 0.8830 | 0.8181 | 0.8493 | 0.4364 | 0.5841 |
| | $c\bar{s}$ | $b\bar{b}$ | $n\bar{b}$ | $s\bar{b}$ | $c\bar{b}$ |
| $\alpha(0)$ | −3.1190 | −19.5854 | −10.2403 | −10.4472 | −12.2572 |
| $\alpha'$ (GeV$^{-2}$) | 0.5692 | 0.2049 | 0.3326 | 0.3277 | 0.2789 |

with the usual understanding on the mixing picture of $\eta$–$\eta'$ ($\omega$–$\phi$).

For these heavy quark states such as the $\eta_b(1S)$, $B_s^*$ and $h_c(1P)$ which are not included in the Meson Summary Table but appear in the Table 13.2 of [2], we want to give some comments. In our analysis of the 1 $^1S_0$ trajectories, we input the mass of the state $\eta_b(1S)$, 9.3 GeV, as the mass of the $b\bar{b}$ state, although the $b\bar{b}$ member of the 1 $^1S_0$ is not well established experimentally. Note

**Table 13-I.** The masses of the $n\bar{n}$, $s\bar{s}$ and $n\bar{s}$ states lying on the 2 $^3S_1$ trajectories

| | $M_{n\bar{n}}$ | | | $M_{s\bar{s}}$ | | | $M_{n\bar{s}}$ | | |
|---|---|---|---|---|---|---|---|---|---|
| Reference | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $j=3$ | $j=5$ |
| Present work | **1.465** | 2.100 | 2.584 | **1.680** | 2.295 | 2.777 | 1.573 | 2.198 | 2.680 |
| Exp. [2] | 1.465 | | | | | | | | |
| [19] | 1.45 | 2.13 | | 1.69 | | | 1.58 | 2.24 | |

**Table 13-II.** The masses of the $c\bar{n}$, $c\bar{s}$, $n\bar{b}$ and $s\bar{b}$ states lying on the 2 $^3S_1$ trajectories

| | $M_{c\bar{n}}$ | | | $M_{c\bar{s}}$ | | | $M_{n\bar{b}}$ | | | $M_{s\bar{b}}$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Reference | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ |
| Present work | 2.588 | 3.181 | 3.680 | 2.690 | 3.279 | 3.777 | 5.813 | 6.309 | 6.769 | 5.910 | 6.406 | 6.864 |
| Exp. [2] | | | | | | | | | | | | |
| [19] | 2.64 | | | 2.73 | | | 5.93 | | | 6.01 | | |
| [20] | 2.629 | | | 2.716 | | | 5.898 | | | 5.984 | | |
| [21] | 2.647 | | | 2.758 | | | 5.924 | | | 6.036 | | |
| [22] | 2.692 | | | 2.806 | | | 5.920 | | | 6.019 | | |
| [23] | 2.62 | 3.19 | | 2.73 | 3.31 | | 5.87 | 6.32 | | 5.97 | 6.42 | |

**Table 13-III.** The masses of the $c\bar{c}$, $b\bar{b}$ and $c\bar{b}$ states lying on the 2 $^3S_1$ trajectories

| | $M_{c\bar{c}}$ | | | $M_{b\bar{b}}$ | | | $M_{c\bar{b}}$ | | |
|---|---|---|---|---|---|---|---|---|---|
| Reference | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ | $J=1$ | $J=3$ | $J=5$ |
| Present work | **3.68596** | 4.263 | 4.770 | **10.02326** | 10.499 | 10.954 | 6.894 | 7.396 | 7.866 |
| Exp. [2] | 3.68596 | | | 10.02326 | | | | | |
| [19] | 3.68 | 4.22 | | 10.00 | 10.45 | | 6.89 | | |
| [23] | 3.73 | 4.24 | | 10.02 | 10.47 | | 6.90 | 7.37 | |
| [24] | 3.686 | | | 10.023 | | | 6.881 | | |
| [25] | 3.719 | 4.332 | | 10.023 | 10.465 | | 6.935 | 7.459 | |
| [26] | 3.686 | | | 10.007 | | | 6.899 | | |
| [27] | 3.690 | | | 10.023 | 10.444 | | 6.921 | | |
| [28] | 3.693 | | | 10.027 | | | 6.914 | | |
| [29] | 3.716 | | | 10.004 | 10.444 | | | | |
| [30] | 3.686 | | | 10.016 | | | 6.886 | | |
| [31] | 3.685 | | | 10.019 | | | | | |
| [32] | 3.670 | | | 10.015 | | | | | |
| [33] | | | | | | | 6.902 | | |
| [34] | | | | | | | 6.917 | | |
| [35] | | | | | | | 6.900 | | |
| [36] | | | | | | | 6.990 | | |

that, as shown in Table 3-III, many theoretical predictions on the mass of the $b\bar{b}$ state of the 1 $^1S_0$ multiplet are in good agreement with the ALEPH measurement that $M_{\eta_b(1S)} = 9300 \pm 20 \pm 20$ MeV [51]. Also, our predicted masses of $B_s$ and $B_c$, which are obtained with the help of $M_{\eta_b(1S)} = 9.3\,\mathrm{GeV}$, are in good agreement with the experimental data and the predictions given by many other references listed in Tables 3-II and 3-III. Therefore, if the state $\eta_b(1S)$ is confirmed experimentally, it would be a good candidate for the $b\bar{b}$ member of the 1 $^1S_0$ multiplet. From Table 5-II, it is clear that the agreement between the theoretical result on the mass of the $c\bar{c}$ state of the 1 $^1P_1$ multiplet and the experimental result that $M_{h_c(1P)} = 3526.14 \pm 0.24\,\mathrm{MeV}$ [2] is good. Also, Table 9-II indicates that many predicted results on the mass of the $s\bar{b}$ state of the 1 $^3S_1$ multiplet are in good agreement with the measured result that $M_{B_s^*} = 5416.6 \pm 3.5\,\mathrm{MeV}$ [2]. Therefore, if the $h_c(1P)$ and $B_s^*$ are confirmed experimentally, the suggestion that the $h_c(1P)$ is the $c\bar{c}$ member of the 1 $^1P_1$ multiplet and the $B_s^*$ is the $s\bar{b}$ member of the 1 $^3S_1$ multiplet seems satisfactory.

Recently, the quantum numbers of the $D_{sJ}(2457)$ were measured by the Belle Collaboration [52]; the mass of the $D_{sJ}(2457)$ was determined to have the value of $2456.5 \pm 1.3 \pm 1.3\,\mathrm{MeV}$, and the measured results on the branching fractions are consistent with the spin-parity assignment for the $D_{sJ}(2457)$ of $1^+$. In the 2004 edition of the Review of Particle Physics [53], this state has been included in the Meson Summary Table[5]. Such experimental results support our prediction that the mass of the $c\bar{s}$ state of the $1\ ^3P_1$ multiplet is about 2.5 GeV.

## 4 Summary and conclusion

In the quasi-linear Regge trajectory ansatz, with the help of additivity of inverse slopes and intercepts, the parameters of the $1\ ^1S_0$, $1\ ^1P_1$, $1\ ^3P_1$, $1\ ^3S_1$, $1\ ^3P_2$ and $2\ ^3S_1$ trajectories are extracted. Based on these parameters, the masses of the states lying on these Regge trajectories mentioned above are estimated. Our predictions are in reasonable agreement with those suggested by many other different approaches. We therefore conclude that the quasi-linear Regge trajectory can, at least at present, give a reasonable description for the meson spectroscopy, and its predictions may be useful for the discovery of the meson states which have not yet been observed.

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[5] In [53], the Particle Data Group use the $D_{sJ}(2460)$ with mass of $2459.3\pm1.3$ MeV for the $c\bar{s}$ state of the $1\ ^3P_1$ multiplet.